329. Proposed by JOHN JAMES QUINN, Ph. D., Scottdale, Pa.

1. Determine the equation of the locus of a fixed point in a circle of radius r, rolling along the axis of an upright cylinder of the same radius, while the axis revolves (carrying the circle with it) through an angle equal to the central angle of the rolling circle formed by the radii to the fixed point and the point of contact.

2. Suppose the point projected into the surface of the cylinder.

3. What is the surface generated by the radius of the rolling circle?

4. What is the surface generated by a radius of the cylinder through the moving point?

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let a=the distance between the axis of the cylinder and the line about which it revolves. Also suppose the plane of the circle is determined by the line a and the axis of the cylinder. Let the circle, in its initial position, be in the xz plane with the point of contact in the same plane.

The position of the moving point at any time, using the axis of the cylinder as initial line and initial point of contact as origin, is $r\theta-r\sin\theta$, and $r(1-\cos\theta)$. We use the double sign, plus when the circle is not between the z-axis and the axis of the cylinder, and the minus when it is.

1. The locus is determined by $x=[a\pm r(1-\cos\theta)]\cos\theta$, $y=[a\pm r(1-\cos\theta)]\sin\theta$, $z=r\theta-r\sin\theta$, θ=central angle of circle.

2. If point is projected in surface of cylinder in plane of circle, the locus is determined by $x=(a\pm r)\cos\theta$, $y=(a\pm r)\sin\theta$, $z=r(\theta-\sin\theta)$.

3. Coordinates of the center of gravity of the moving radius of the circle are, $x=[a\pm r(1-\frac{1}{2}\cos\theta)]\cos\theta$, $y=[a\pm r(1-\frac{1}{2}\cos\theta)]\sin\theta$, $z=r(\theta-\frac{1}{2}\sin\theta)$.

$\therefore$ Surface=rS where S is given as follows:

$$S=\int_0^\theta [4a^2\pm 8ar+8r^3\mp 4ar\cos\theta(4\sin^2\theta-1)+r^2(1-16\cos\theta)(1+\cos^2\theta)]^{\frac{1}{2}}\,]d\theta.$$

4. Coordinates of center of gravity of radius of cylinder through moving point are $x=(a\pm\frac{1}{2}r)\cos\theta$, $y=(a\pm\frac{1}{2}r)\sin\theta$, $z=r(\theta-\sin\theta)$.

$\therefore$ Surface=rS where

$$S=\int_0^\theta [(a\pm\tfrac{1}{2}r)^2+r^2(1-\cos\theta)^2]^{\frac{1}{2}}\,d\theta.$$

Also solved by H. V. Spunar.

CALCULUS.

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257. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

If $A=\displaystyle\int_0^\infty \frac{dx}{\sqrt{x}\,(2a+x)^n}$, $B=\displaystyle\int_0^\infty \frac{y^n dy}{\sqrt{y}\,(a^2+y^2)^n}$, find A/B.

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and the PROPOSER.

Let $x=2a\tan^2\theta$, $y=a\tan\phi$, $m=\frac{1}{2}(2n-1)$.

$$\therefore A=\frac{2}{(2a)^{n-\frac{1}{2}}}\int_0^{\frac{1}{2}\pi}\cos^{2(n-1)}\theta\, d\,\theta=\frac{1}{2(2a)^m}\int_0^{\frac{1}{2}\pi}\cos^{2m-1}\theta\, d\,\theta=\frac{\sqrt{(\pi)}\Gamma(m)}{(2a)^m\Gamma\left(\frac{2m+1}{2}\right)}.$$

$$B=\frac{1}{a^{n-\frac{1}{2}}}\int_0^{\frac{1}{2}\pi}\tan^{n-\frac{1}{2}}\phi\cos^{2(n-1)}\phi\, d\phi=\frac{1}{a^m}\int_0^{\frac{1}{2}\pi}\sin^m\phi\cos^{m-1}\phi\, d\,\phi$$

$$=\frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{m}{2}\right)}{2a^m\ \Gamma\left(\frac{2\,m+1}{2}\right)}.$$

But $\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{m}{2}\right)=\frac{\sqrt{(\pi)}\Gamma(m)}{2^{m-1}}$.

$$\therefore B=\frac{\sqrt{(\pi)}\Gamma(m)}{(2a)^m\Gamma\left(\frac{2\,m+1}{2}\right)}=A.\quad \text{Hence } A/B=1.$$

Exhaustive solutions, though differing in results from the one given here, were received from M. V. Spunar, Francis Rust, and T. G. Wodo.

258. Proposed by A. H. HOLMES, Brunswick, Maine.

Evaluate $\displaystyle\int_0^{\frac{1}{2}a}\frac{dx}{\sqrt{[2ax-x^2\sqrt{(a^2-x^2)}]}}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $x=x\sin\theta$. Then

$$I=\int_0^{\frac{1}{2}a}\frac{dx}{\sqrt{[2ax-x^2\sqrt{(a^2-x^2)}]}}=\int_0^{\frac{1}{6}\pi}\frac{\cos\theta\, d\,\theta}{\sqrt{[2\sin\theta-a\sin^2\theta\cos\theta]}}.$$

$$\therefore I=\frac{1}{\sqrt{2}}\int_0^{\frac{1}{6}\pi}\left[1+\frac{a}{4}\sin\theta\cos\theta+\frac{3a^2}{32}\sin^2\theta\cos^2\theta+\frac{5a^3}{128}\sin^3\theta\cos^3\theta\right.$$

$$\left.+\frac{35a^4}{2048}\sin^4\theta\cos^4\theta+\frac{64a^5}{8192}\sin^5\theta\cos^5\theta+\frac{231a^6}{65536}\sin^6\theta\cos^6\theta+\dots\right]\frac{\cos\theta\, d\,\theta}{\sqrt{(\sin\theta)}}$$

$$=1+\frac{31}{15}\cdot\frac{a^2}{2^9}+\frac{85015}{1989}\cdot\frac{a^4}{2^{19}}+\frac{2350494}{38675}\cdot\frac{a^6}{2^{28}}+\dots+\frac{1}{\sqrt{2}}\int_0^{\frac{1}{6}\pi}\left[\frac{a}{4}\sin\theta\cos\theta\right.$$